Assessing the Usefulness of Simple Mathematical Models to Describe Soil Carbon Dynamics

Armen R. Kemanian
Biological Systems Engineering Dept. – Washington State University
Valipuram S. Manoranjan
Department of Mathematics – Washington State University
David R. Huggins
USDA-ARS Pullman WA
Claudio O. Stöckle
Biological Systems Engineering Dept. – Washington State University

Outline

- Introduction: why simple models?
- Objectives.
- Hénin and Dupuis (1945) formulation and applications.
- Three new models: development and theoretical behavior.
- Testing with long-term data.
- Concluding remarks.

Objective

Develop analytical solutions to soil carbon temporal dynamics based on a minimum set of assumptions.

A word from Monteith:

"…complexity (*in models*) is rarely achieved without recourse to chains of assumptions …"

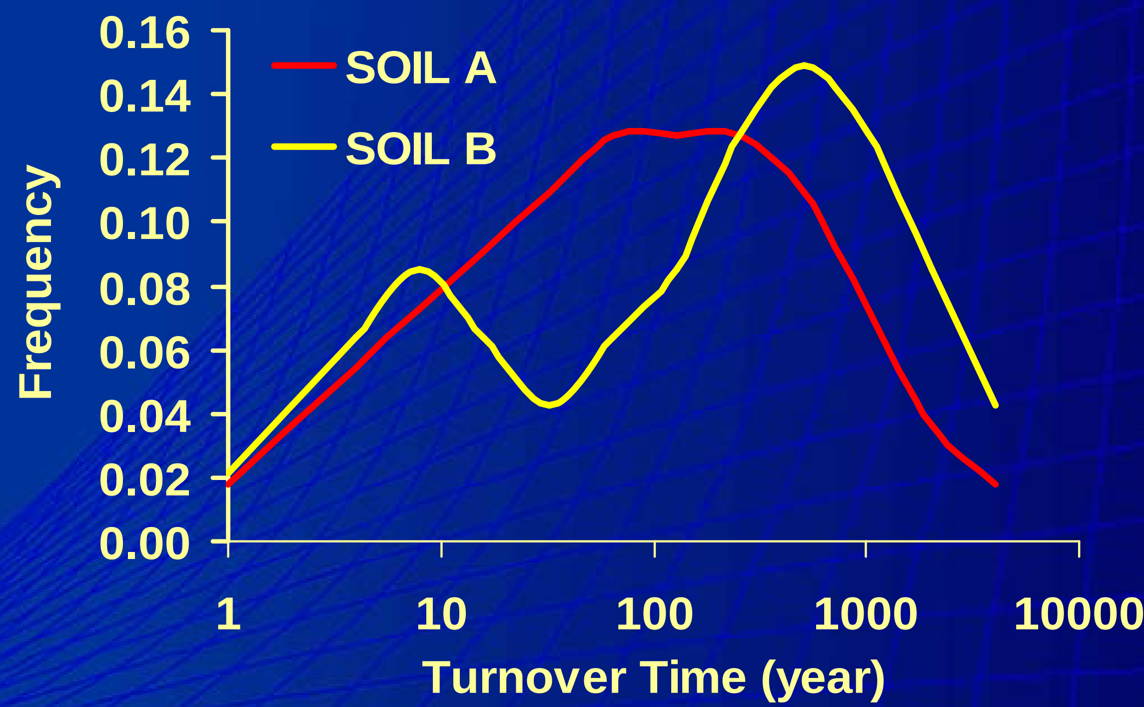
"The tendency for models to become more complex should be balanced by attempts to identify and eliminate inputs or relationships that turn out to have little bearing on the output."

"I believe there is also a place for relatively simple analytical models …"

In Proceedings of 11th Congress ISSS, 1978 (page 385)

Why simple models?

- Soil organic matter is composed of different fractions with varying (continuum) turnover rates.
- At best, SOM is treated as composed of discrete fractions with distinct properties.



Why simple models?

- SOM reaches steady state condition gradually.
- Mechanistic simulation models tend to give a smooth response to factors affecting SOM dynamics.
- Can models be further simplified when the interest is in the long-term C evolution?

Hénin and Dupuis (1945)

$$dC_s/dt = hC_i - kC_s$$

C_s is the soil organic Carbon (Mg ha^{-1})

t is time (year)

h is the humification constant

C_i is the carbon input

k is the apparent soil decomposition rate

At steady state: $C_s = C_i h/k$

Andrén and Kätterer (1997)

$$dC_y/dt = C_i - r_e k_y C_y$$

$$dC_o/dt = r_e h k_y C_y - k_o C_o$$

r_e is a factor accounting for environmental effects

"*y*" subscript indicates young organic matter

"*o*" subscript indicates old organic matter

k_y = 0.8 yr^{-1}; k_o = 0.006 yr^{-1};

h = 0.12 – 0.31; C_y = 3 Mg C ha^{-1}

Alternatives to Hénin and Dupuis (1945)

(1) Assume k varies as a function of C_s (the higher C_s the higher k)

(2) Assume h varies as a function of C_s (the higher C_s the lower h)

(3) Assume both k and h are a function of C_s (1 & 2)

For simplicity, we assumed that both dependencies are linear on C_s

$k = f(C_s)$

$$k(C_s) = k_n(1 + C_s/C_k)$$

$$dC_s/dt = hC_i - k_n(1 + C_s/C_k)C_s$$

k_n **is the minimum apparent decomposition rate**

C_k **is a soil dependent** C_s **content**

$k = f(C_s)$

$$C_s(t) = C_k(a_2 A\exp(-k_n(a_2 - a_1)t - a_1)/(1 - A\exp(-k_n(a_2 - a_1)t))$$

$$a_1 = -0.5(1 + (1 + 4b)^{1/2})$$

$$a_2 = 0.5((1 + 4b)^{1/2} - 1)$$

$$b = hC_i / (k_n C_k)$$

A **is an integration constant**

At steady state: $C_s = 0.5C_k (1 + (1 + 4b)^{1/2})$

$$h = f(C_s)$$

$$h(C_x) = h_x(1 - C_s/C_x)$$

$$dC_s/dt = h_x(1 - C_s/C_x)C_i - kC_s$$

h_x **is the maximum humification rate**

C_x **is the maximum soil carbon carrying capacity**

$h = f(C_s)$

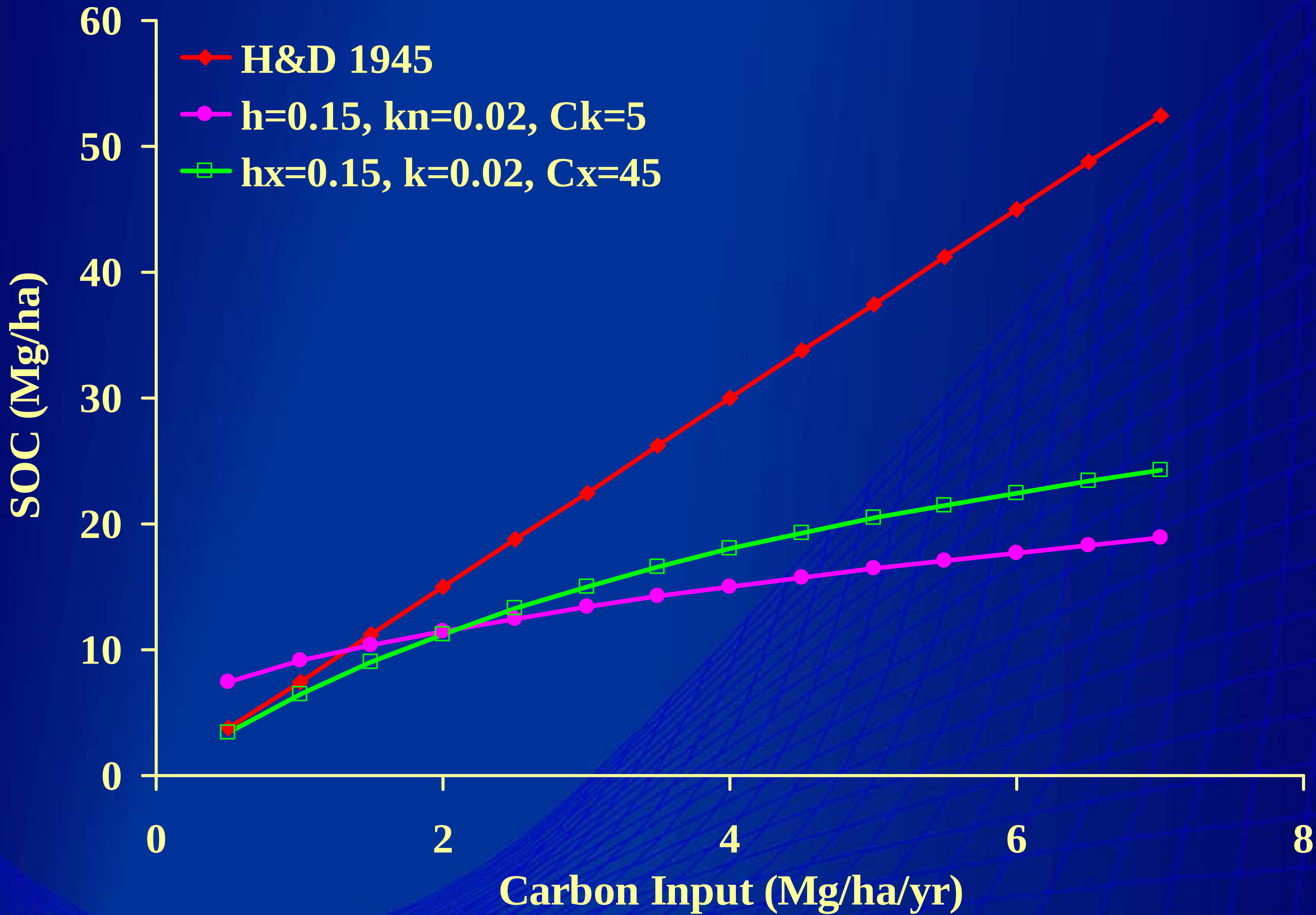
$$C_s(t) = h_xC_i/c + (C_o - h_xC_i/c)exp(-ct)$$

$$c = h_xC_i/C_x + k$$

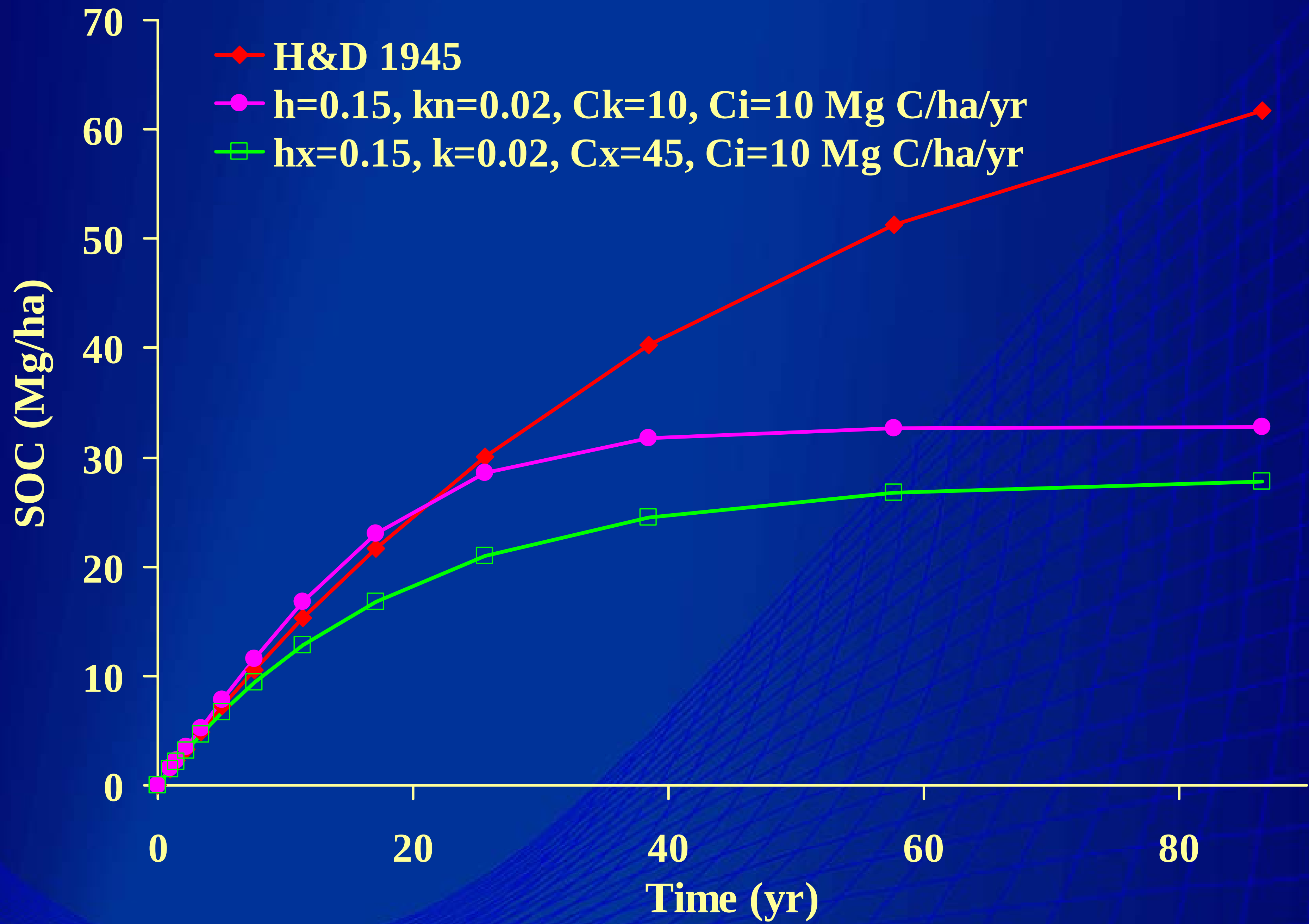
h_x **is the maximum humification**

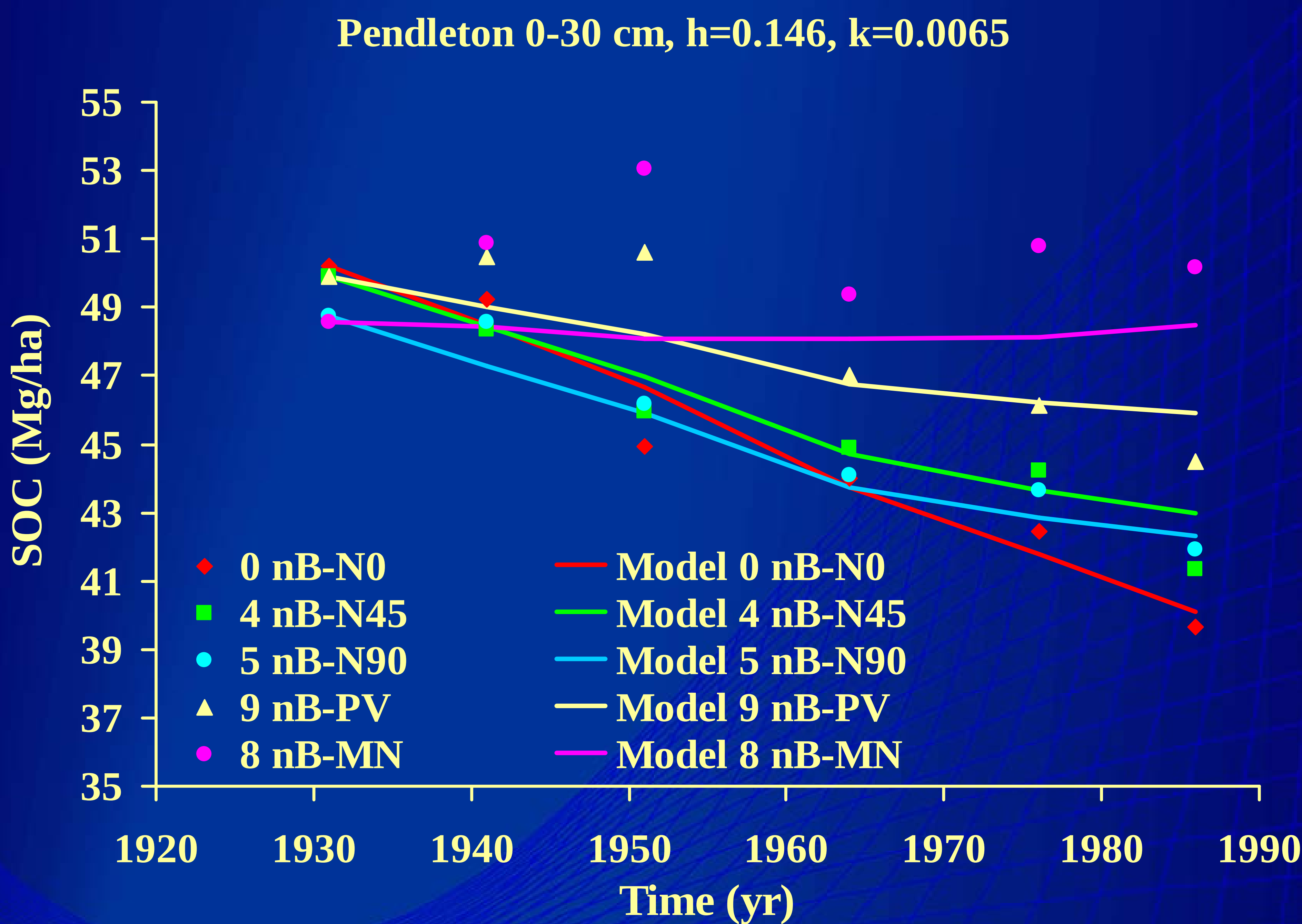
C_x **is the maximum soil carbon carrying capacity**

At steady state: $C_s = h_xC_iC_x/(h_xC_i + kC_x)$

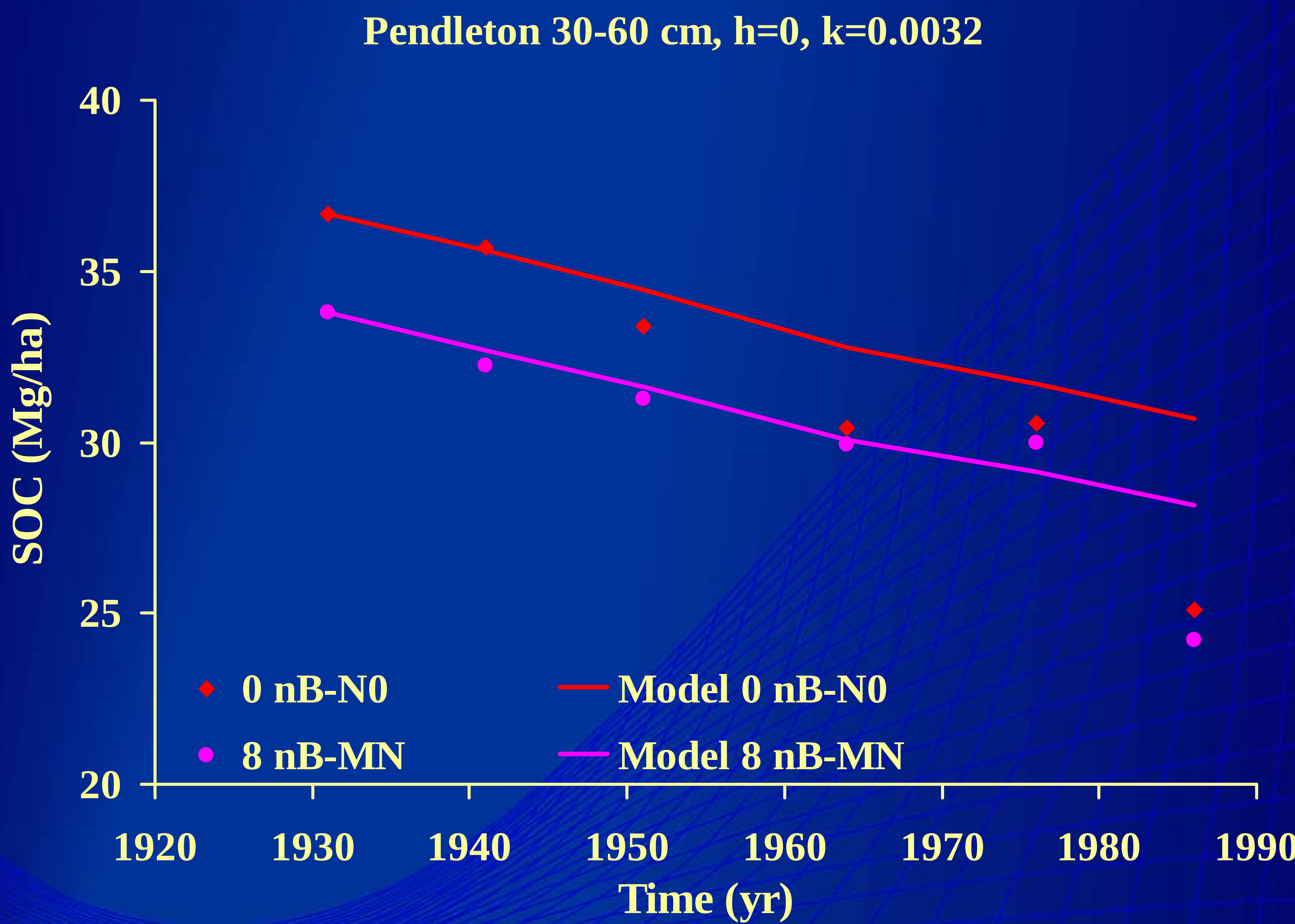


H&D 1945
h=0.15, kn=0.02, Ck=5
hx=0.15, k=0.02, Cx=45
SOC (Mg/ha)
60
50
40
30
20
10
0
0
2
4
6
8
Carbon Input (Mg/ha/yr)

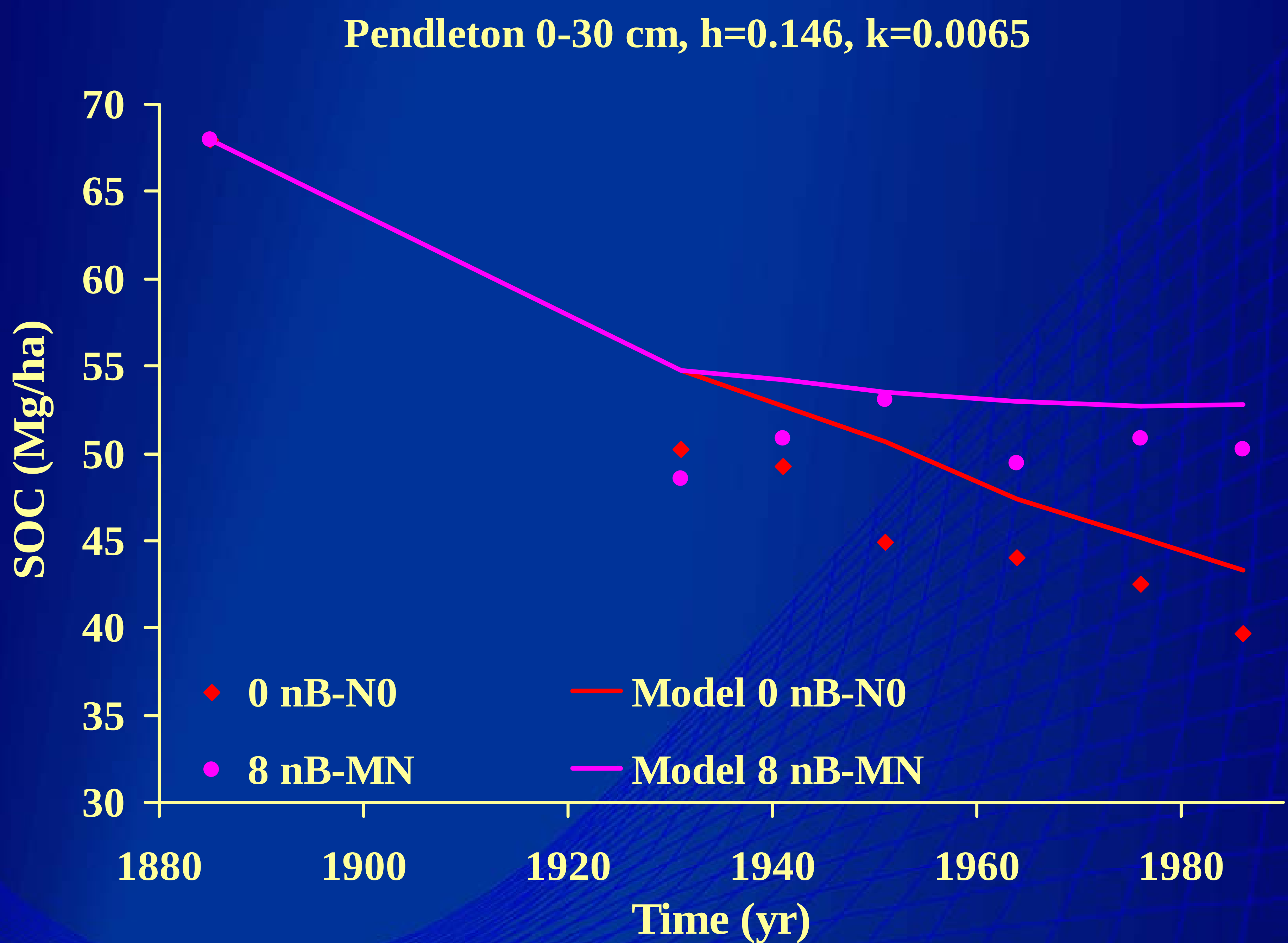

H&D 1945
h=0.15, kn=0.02, Ck=10, Ci=10 Mg C/ha/yr
hx=0.15, k=0.02, Cx=45, Ci=10 Mg C/ha/yr
SOC (Mg/ha)
Time (yr)
0
10
20
30
40
50
60
70
0
20
40
60
80



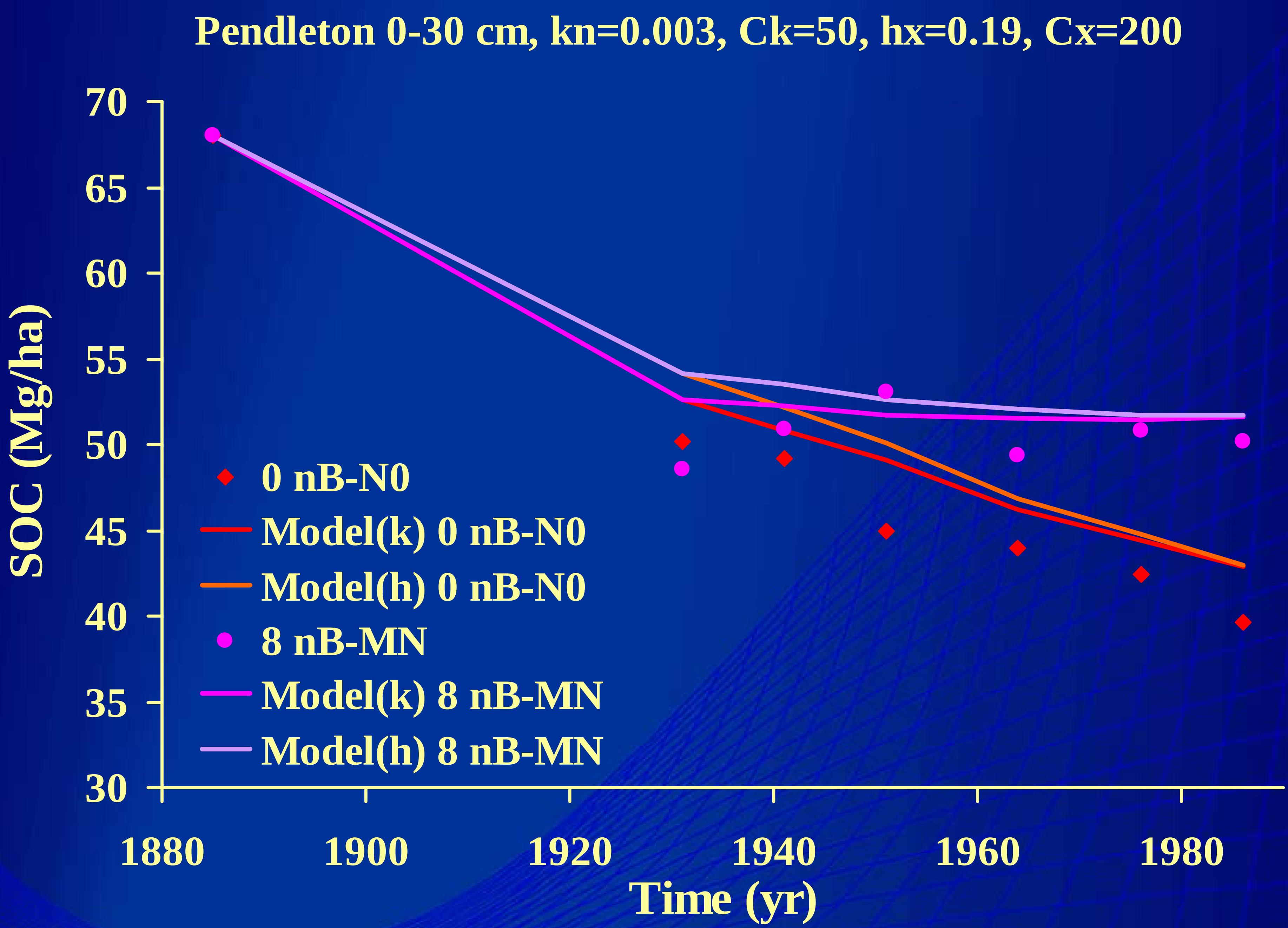
Pendleton 0-30 cm, h=0.146, k=0.0065
SOC (Mg/ha)
55
53
51
49
47
45
43
41
39
37
35
1920
1930
1940
1950
1960
1970
1980
1990
Time (yr)
0 nB-N0
4 nB-N45
5 nB-N90
9 nB-PV
8 nB-MN
Model 0 nB-N0
Model 4 nB-N45
Model 5 nB-N90
Model 9 nB-PV
Model 8 nB-MN

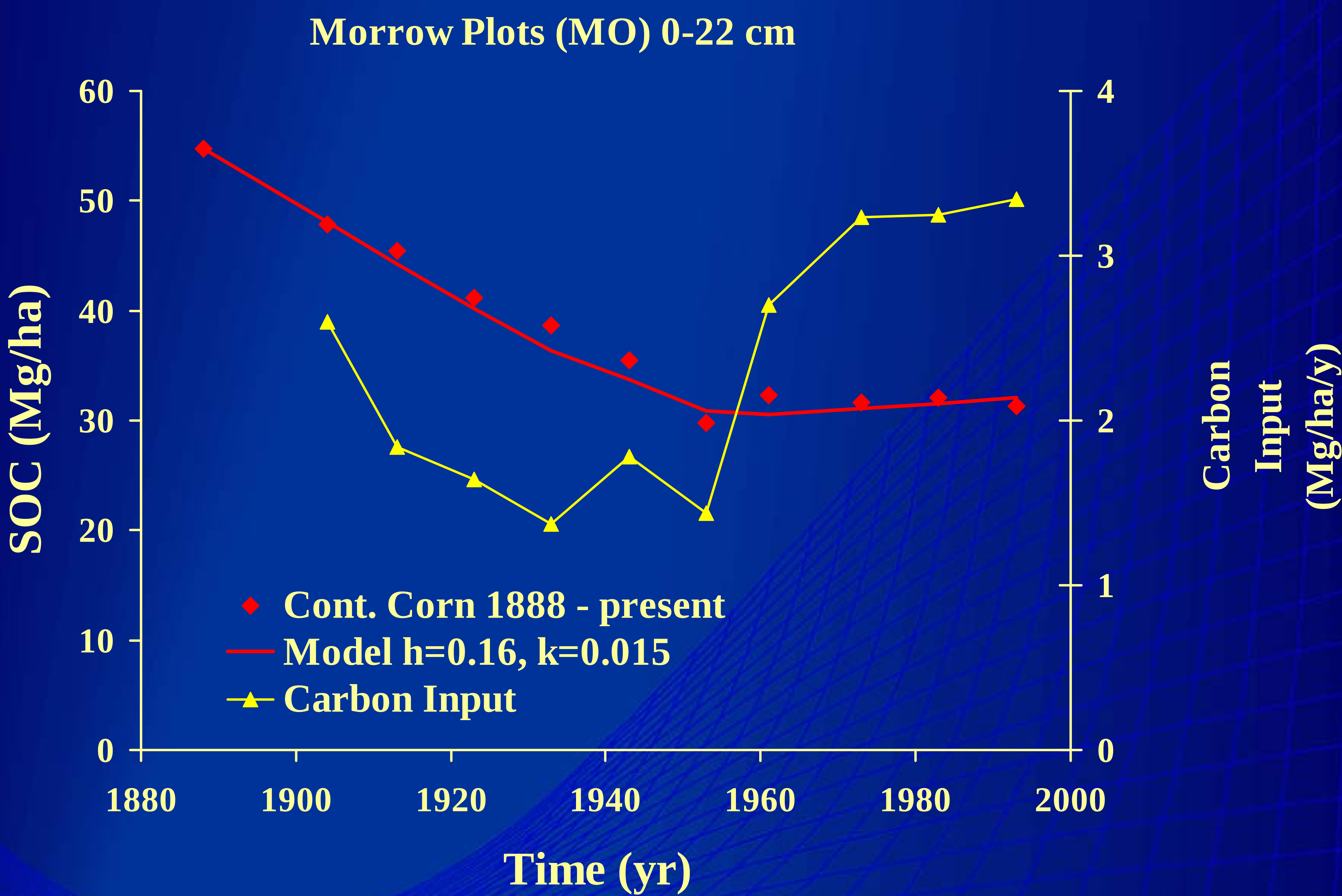


Pendleton 30-60 cm, h=0, k=0.0032
40
35
30
25
20
SOC (Mg/ha)
1920
1930
1940
1950
1960
1970
1980
1990
Time (yr)
0 nB-N0
8 nB-MN
Model 0 nB-N0
Model 8 nB-MN

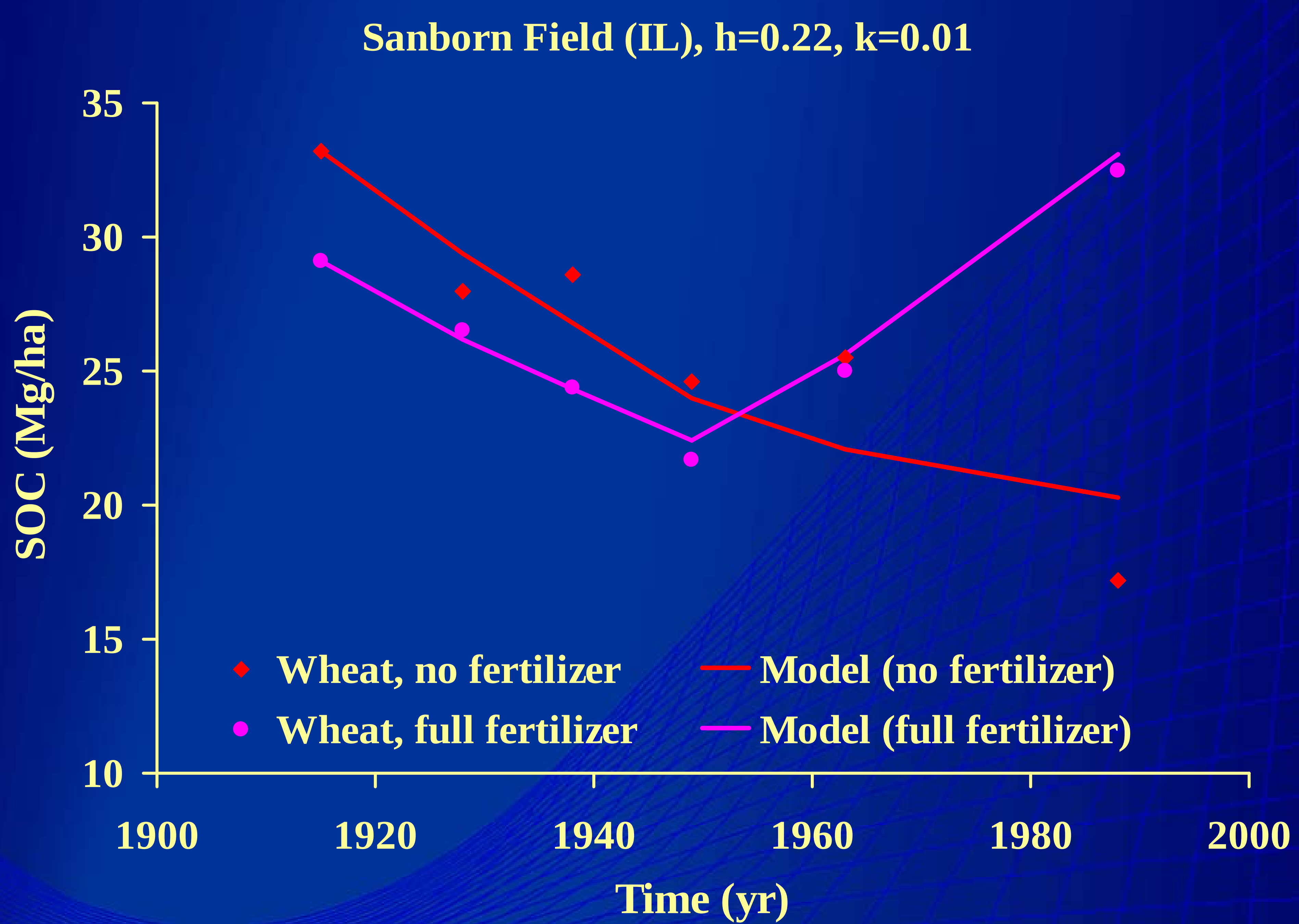


Pendleton 0-30 cm, h=0.146, k=0.0065
SOC (Mg/ha)
70
65
60
55
50
45
40
35
30
1880
1900
1920
1940
1960
1980
Time (yr)
0 nB-N0
8 nB-MN
Model 0 nB-N0
Model 8 nB-MN


Pendleton 0-30 cm, kn=0.003, Ck=50, hx=0.19, Cx=200
SOC (Mg/ha)
70
65
60
55
50
45
40
35
30
1880
1900
1920
1940
1960
1980
Time (yr)
0 nB-N0
Model(k) 0 nB-N0
Model(h) 0 nB-N0
8 nB-MN
Model(k) 8 nB-MN
Model(h) 8 nB-MN



Morrow Plots (MO) 0-22 cm
SOC (Mg/ha)
60
50
40
30
20
10
0
4
3
2
1
0
Carbon Input (Mg/ha/y)
Cont. Corn 1888 - present
Model h=0.16, k=0.015
Carbon Input
1880
1900
1920
1940
1960
1980
2000
Time (yr)


Sanborn Field (IL), h=0.22, k=0.01
35
30
25
20
15
10
SOC (Mg/ha)
1900
1920
1940
1960
1980
2000
Time (yr)
Wheat, no fertilizer
Wheat, full fertilizer
Model (no fertilizer)
Model (full fertilizer)

Concluding remarks

- **The simple Hénin and Dupuis (1945) model fits relatively well the cases analyzed.**
- **The adjustments of *k* or *h* provide more flexibility to the model, but data availability and quality prevent being conclusive until further analysis.**
- **The adjustments of *k* or *h* can be non-linear, but analytical solutions may not be possible.**
- **The adjustments can also be applied in mechanistic simulation models.**