Assessing the Usefulness of Simple Mathematical Models to Describe Soil Carbon Dynamics

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Outline

• Introduction: why simple models?
• Objectives.
• Hénin and Dupuis (1945) formulation and applications.
• Three new models: development and theoretical behavior.
• Testing with long-term data.
• Concluding remarks.
Objective

Develop analytical solutions to soil carbon temporal dynamics based on a minimum set of assumptions.
A word from Monteith:

“...complexity (in models) is rarely achieved without recourse to chains of assumptions ...”

“The tendency for models to become more complex should be balanced by attempts to identify and eliminate inputs or relationships that turn out to have little bearing on the output.”

“I believe there is also a place for relatively simple analytical models ...”

In Proceedings of 11th Congress ISSS, 1978 (page 385)
Why simple models?

- Soil organic matter is composed of different fractions with varying (continuum) turnover rates.
- At best, SOM is treated as composed of discrete fractions with distinct properties.
Why simple models?

- SOM reaches steady state condition gradually.
- Mechanistic simulation models tend to give a smooth response to factors affecting SOM dynamics.
- Can models be further simplified when the interest is in the long-term C evolution?
Hénin and Dupuis (1945)

\[ \frac{dC_s}{dt} = hC_i - kC_s \]

- \( C_s \) is the soil organic Carbon (Mg ha\(^{-1}\))
- \( t \) is time (year)
- \( h \) is the humification constant
- \( C_i \) is the carbon input
- \( k \) is the apparent soil decomposition rate

At steady state: \( C_s = \frac{C_i h}{k} \)
Andrén and Kätterer (1997)

\[ \frac{dC_y}{dt} = C_i - r_e k_y C_y \]
\[ \frac{dC_o}{dt} = r_e h k_y C_y - k_o C_o \]

\( r_e \) is a factor accounting for environmental effects

“\( y \)” subscript indicates young organic matter

“\( o \)” subscript indicates old organic matter

\( k_y = 0.8 \text{ yr}^{-1}; \ k_o = 0.006 \text{ yr}^{-1}; \)

\( h = 0.12 - 0.31; \ C_y = 3 \text{ Mg C ha}^{-1} \)
Alternatives to Hénin and Dupuis (1945)

(1) Assume $k$ varies as a function of $C_s$ (the higher $C_s$ the higher $k$)

(2) Assume $h$ varies as a function of $C_s$ (the higher $C_s$ the lower $h$)

(3) Assume both $k$ and $h$ are a function of $C_s$ (1 & 2)

For simplicity, we assumed that both dependencies are linear on $C_s$
$k = f(C_s)$

$k(C_s) = k_n(1 + C_s/C_k)$

$\frac{dC_s}{dt} = hC_i - k_n(1 + C_s/C_k)C_s$

$k_n$ is the minimum apparent decomposition rate

$C_k$ is a soil dependent $C_s$ content
\[ k = f(C_s) \]

\[ C_s(t) = C_k(a_2 A \exp(-k_n(a_2 - a_1)t - a_1)/(1 - A \exp(-k_n(a_2 - a_1)t)) \]

\[ a_1 = -0.5(1 + (1 + 4b)^{1/2}) \]

\[ a_2 = 0.5((1 + 4b)^{1/2} - 1) \]

\[ b = hC_i / (k_n C_k) \]

*A* is an integration constant

At steady state: \[ C_s = 0.5C_k (1 + (1 + 4b)^{1/2}) \]
\[ h = f(C_s) \]

\[ h(C_x) = h_x(1 - C_s/C_x) \]

\[ \frac{dC_s}{dt} = h_x(1 - C_s/C_x)C_i - kC_s \]

\[ h_x \] is the maximum humification rate

\[ C_x \] is the maximum soil carbon carrying capacity
\[ h = f(C_s) \]

\[ C_s(t) = h_x C_i/c + (C_o - h_x C_i/c) \exp(-ct) \]

\[ c = h_x C_i/C_x + k \]

\( h_x \) is the maximum humification

\( C_x \) is the maximum soil carbon carrying capacity

At steady state: \[ C_s = h_x C_i C_x / (h_x C_i + k C_x) \]
H&D 1945

$H = 0.15$, $k_n = 0.02$, $C_k = 5$

$H_x = 0.15$, $k = 0.02$, $C_x = 45$
H&D 1945

- $h=0.15$, $kn=0.02$, $C_k=10$, $C_i=10$ Mg C/ha/yr
- $hx=0.15$, $k=0.02$, $C_x=45$, $C_i=10$ Mg C/ha/yr
### Pendleton 0-30 cm, h=0.146, k=0.0065

<table>
<thead>
<tr>
<th>SOC (Mg/ha)</th>
<th>Model 0 nB-N0</th>
<th>Model 4 nB-N45</th>
<th>Model 5 nB-N90</th>
<th>Model 9 nB-PV</th>
<th>Model 8 nB-MN</th>
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<tbody>
<tr>
<td>0 nB-N0</td>
<td>51</td>
<td>50</td>
<td>49</td>
<td>47</td>
<td>46</td>
</tr>
<tr>
<td>4 nB-N45</td>
<td>49</td>
<td>48</td>
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</tr>
<tr>
<td>5 nB-N90</td>
<td>47</td>
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<td>9 nB-PV</td>
<td>45</td>
<td>44</td>
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<td>41</td>
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<tr>
<td>8 nB-MN</td>
<td>43</td>
<td>42</td>
<td>41</td>
<td>40</td>
<td>39</td>
</tr>
</tbody>
</table>

**Graph:**
- **SOC (Mg/ha)** vs **Time (yr)**
- **Models**:
  - Model 0 nB-N0
  - Model 4 nB-N45
  - Model 5 nB-N90
  - Model 9 nB-PV
  - Model 8 nB-MN

**Legend:**
- 0 nB-N0
- 4 nB-N45
- 5 nB-N90
- 9 nB-PV
- 8 nB-MN

**Time (yr):**
- 1920
- 1930
- 1940
- 1950
- 1960
- 1970
- 1980
- 1990
Pendleton 30-60 cm, h=0, k=0.0032

SOC (Mg/ha)

0 nB-N0

8 nB-MN

Time (yr)

Pendleton 0-30 cm, h=0.146, k=0.0065

SOC (Mg/ha)

- 0 nB-N0
- 8 nB-MN
- Model 0 nB-N0
- Model 8 nB-MN

Time (yr)

1880 1900 1920 1940 1960 1980
Pendleton 0-30 cm, $kn=0.003$, $Ck=50$, $hx=0.19$, $Cx=200$
Morrow Plots (MO) 0-22 cm

![Graph showing soil organic carbon (SOC) and carbon input over time.]

- **SOC (Mg/ha)**: The graph displays the soil organic carbon content over time, measured in megagrams per hectare.
- **Carbon Input (Mg/ha/y)**: The graph also shows the carbon input over time, measured in megagrams per hectare per year.
- **Time (yr)**: The x-axis represents years, ranging from 1880 to 2000.

**Key Graph Elements**:
- **Red Dots**: Continuous Corn from 1888 to present.
- **Red Line**: Model with parameters $h=0.16$ and $k=0.015$.
- **Yellow Triangles**: Carbon Input.

**Interpretation**:
- The red dots represent the continuous corn application from 1888 to the present, indicating a steady decline in soil organic carbon due to high tillage and recurrent corn farming.
- The red line, representing a model with parameters $h=0.16$ and $k=0.015$, suggests a more controlled path of carbon loss, following the actual data closely.
- The yellow triangles indicate the carbon input over time, showing fluctuations that could be related to agricultural practices and environmental factors.

These graphs highlight the impact of agricultural practices on soil organic carbon levels and the importance of understanding and possibly controlling these changes to maintain soil health and fertility.
Concluding remarks

• The simple Hénin and Dupuis (1945) model fits relatively well the cases analyzed.

• The adjustments of $k$ or $h$ provide more flexibility to the model, but data availability and quality prevent being conclusive until further analysis.

• The adjustments of $k$ or $h$ can be non-linear, but analytical solutions may not be possible.

• The adjustments can also be applied in mechanistic simulation models.