Stochastic Simulation and Data Assimilation to Estimate Soil Carbon Content

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Introduction

- Accurate monitoring methods are needed if soil C sequestration is to be an accepted method for offsetting CO_2 emissions.
- It is not practical to estimate soil C dynamics using measurements only (Izaurralde et al., 1998).
- Both simulation and measurement-based estimates are uncertain.
- Stochastic simulation can help quantify uncertainty (Ogle et al., 2003) .

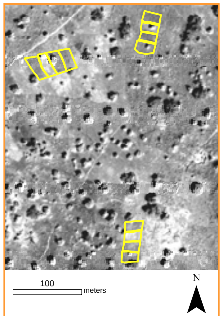
Hypothesis

- The Extended Kalman Filter (ExKF; Graham, 2002) data assimilation algorithm is useful to combine stochastic simulations and measurements to improve accuracy and reduce uncertainty in soil C estimates.

Materials and Methods

Test Site

- The ExKF algorithm was tested for estimation of soil C in 12 fields in Wa, Ghana over a 20-year period.

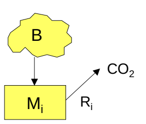


September 2004 QuickBird Image of 12 test fields near Wa, Ghana.

Stochastic Simulation

- For this test, we used a simple one-pool model (Jones, 2004) for the soil C in the top 20 cm of soil (see below)
- M (kg C ha^{-1}) is the soil C in a field and R (yr^{-1}) is the decomposition rate parameter for M.
- Both M and R are random variables.

Diagram



System Model

$$M_{k+1,i} = (1 - R_i)M_{k,i} + B_{K+1,i} + (\eta)_{K+1,i}$$

$$M_{k+1,i} = \overline{M}_{k+1,i} + (\varepsilon_M)_{K+1,i}$$

$$R_i = \overline{R}_i + \varepsilon_{Ri}$$

- Propagation equations for the expected values ($\overline{M}$ and $\overline{R}$) and covariances (P) of the random variables of the model are derived using properties of random variables (see below).

Propagation Equations

$$\overline{M}_{K+1,i} = (1 - \overline{R}_i)\overline{M}_{K,i} + B_{K+1,i} - (P_{MiRi})_K$$

$$(P_{MiMj})_{K+1} = (P_{MiMj})_K(1 - \overline{R}_i)(1 - \overline{R}_j) - (P_{MiRj})_K(1 - \overline{R}_i)\overline{M}_{K,j} - (P_{MjRi})_K(1 - \overline{R}_j)\overline{M}_{K,i} + (P_{RiRj})_K M_{K,i}, M_{K,j} + E[\eta_K \eta_K]$$

$$(P_{MiRi})_{K+1} = E[(M_{K+1,i} - \overline{M}_{K+1,i})(R_j - \overline{R}_j)] = (1 - \overline{R}_j)(P_{MiRj})_K - (P_{RiRj})_K M_{K,i}$$

$$(P_{RiRj})_{K+1} = E[(R_{K+1,i} - \overline{R}_{K+1,i})(R_{K+1,j} - \overline{R}_{K+1,j})] = (P_{RiRj})_K$$

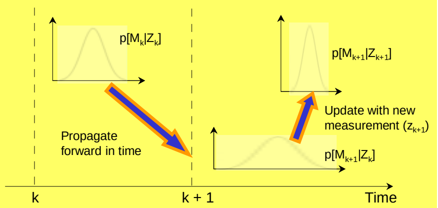
ExKF Details

- When a measurement is made, the vector containing values of $\overline{M}$ and $\overline{R}$ for each field ($\hat{x}(-)$), which are conditioned on previous measurements, is updated to $\hat{x}(+)$.

$$\hat{X}_{K+1}(+) = \hat{X}_{K+1}(-) + K_{K+1}V_{K+1}$$

$$K_{K+1} = P_{K+1}(-)H^T_{K+1}\left[H_{K+1}P_{K+1}(-)H^T_{K+1} + W\right]^{-1}$$

- The main factors affecting the updated states are the differences between measured and simulated states, expressed in the V vector, the variance and covariance of estimates, expressed in the K matrix, and the measurement variance matrix, W.
- Spatial covariance between model states is critical for the ExKF to update states at unmeasured locations.



Evolution of conditional probability density (adapted from Margulis et al., 2002)

- The ExKF is based on Bayes' Rule of conditional probability. The sketch above illustrates the effect of using the ExKF to condition an estimate of a random variable with a measurement.

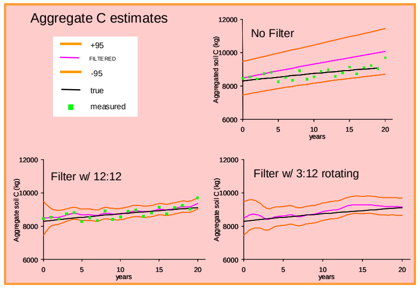
Experiment Details

- Geostatistical analysis with soil C measurements from 2003 showed spatial correlation up to 53 m.
- A semivariogram model developed from the 2003 data was used with the Unconditional Sequential Gaussian Simulation (Goovaerts, 1997) algorithm to initialize expected values of M, R and their covariance matrix P.

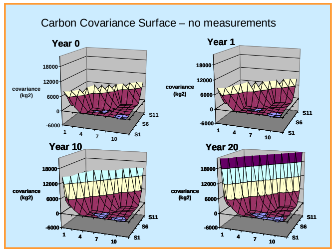
- The ExKF was tested using a generated 20-year true and measurement time series of soil C in each field.
- In order to test the updating capability of the ExKF, the expected value of R was initialized with biased-low values.
- Three measurement scenarios were investigated:
  - ✓ No filter (no measurement updates)
  - ✓ 12 of 12 fields measured annually
  - ✓ 3 of 12 fields (rotating) measured annually

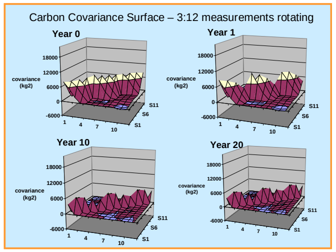
Results and Discussion

- The expected value in the No-Filter case diverges from the 'true' value.
- The 3 of 12 rotating-measurement case compares well with the 12 of 12 measurement case with 75% fewer measurements.
- Measurements in the 3 of 12 rotating case were chosen so all fields were within the correlation range of at least one measurement.



- The effect of the ExKF in reducing the estimate variance can be seen from the graphs above and below.
- Further work is needed with actual long-term spatiotemporal datasets and more realistic soil C models.
- Another area that needs further research is the estimation of actual change in soil C, rather than accurate simulation of the soil C content at a given time.





Summary

- The ExKF combined measurements with stochastic simulation to improve aggregate estimates of soil C content.
- The 'smart' sampling scheme, which sampled a 3 different fields each year, each within the range of correlation of other fields, provided good results with with 75% fewer measurements than the most intensive measurement scenario.
- Areas for further research include:
  - ✓ application of the ExKF with more realistic soil C models,
  - ✓ application with real long-term spatiotemporal datasets, and
  - ✓ estimation of changes in soil C at a site.

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